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Time-dependent AdS/CFT duality and null singularity

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Abstract: We consider AdS/CFT correspondence for time-dependent IIB backgrounds in this paper. The supergravity solutions we construct are supersymmetric pp-waves on AdS and may have null singularity in the bulk. The dual gauge theory is also constructed explicitly and is given by a time-dependent supersymmetric Yang-Mills theory living on the boundary. Apart from the usual terms that are dictated by the geometry, our gauge theory action features also a time-dependent axion coupling and a time-dependent gauge coupling. Both of which are necessary due to the presence of a nontrivial dilaton and axion profile in the supergravity solution. The proposal is supported by a precise matching in the symmetries and functional dependence on the null coordinate of the two theories. As applications, we show how the bulk Einstein equation may be reproduced from the gauge theory. We also study and compare the behaviour of the field theory two-point functions. We find that the two-point function computed by using duality is different from that by doing a direct field theory computation. In particular the spacetime singularity is not seen in our gauge theory result, suggesting that the spacetime singularity may be resolved in the gauge theory.

Keywords: Gauge-gravity correspondence, AdS-CFT Correspondence, Space-Time Symmetries.

Contents

1. Introduction

The understanding of the fundamental nature and quantum properties of spacetime is one of the most important questions for theoretical high energy physics. String theory is a consistent theory of quantum gravity. As such, string theory should be able to provide a satisfactory resolution for outstanding problems such as the entropy of a black hole, spacelike singularity inside the horizon of a black hole and the big bang singularity in the early universe. To achieve this, it is necessary to understand string theory on time-dependent backgrounds [1]. Unfortunately time dependent backgrounds are difficult to work with in general. An exceptional class of models which is simple enough to work with and yet may provide a useful approach is the time-dependent orbifold model discussed first by Horowitz and Steif [2]. In particular, the singularity properties of the string S-matrix and the stability of the background against gravitational backreaction has been examined understanding and treatment of the gravitational backreaction is crucial to establish reliable string results. An understanding of string theory in time-dependent background is still beyond the scope of traditional perturbative string theory.

Recently, powerful nonperturbative formulations of string theory, namely matrix theory [9, 10] and AdS/CFT correspondence [11, 12], have been proposed and put forward. It is natural to apply these ideas in the studies of time-dependent backgrounds, and try to say something about the nature and fate of the spacetime singularity. Holographic description of time-dependent backgrounds via the AdS/CFT correspondence was considered in,

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for example, more recently in [17] where a big crunch cosmology was constructed and the dual field theory description was examined. In an interesting proposal [18], a simple supersymmetric time-dependent background with a null singularity is the holographic dual of a matrix string theory. The time-dependence of the background is proposed to be encoded through the time-dependent coupling of the gauge theory. Extensions and related works can be found in [19]. Holographic description of cosmology in terms of matrix theory can be found in [20].

As is usual in AdS/CFT correspondence, supersymmetry can be expected to play an important role. The existence of supersymmetry allows us a better control over the string/supergravity background and over the quantum and nonperturbative behaviour of the field theory. In the construction of [17], the supergravity solution was constructed for a class of boundary conditions preserving the asymptotically AdS symmetries. However supersymmetry is not preserved in that construction. On the other hand, in the proposal [18], although the background is half BPS, the supersymmetry of the dual matrix string theory is broken due to the presence of finite lightcone momentum.

Supersymmetric time-dependent background has been considered in earlier works. In particular in [13], a time-dependent deformation of the pp-wave geometry has been constructed and the AdS/CFT correspondence considered. Although the background in these works has pp-singularities [21], these singularities are situated at the horizon ($u = \infty$) and as a result one can expect that their effects will be red-shifted away and will not show up in the dual field theory [22]. This is indeed the case as confirmed by the calculations of [13]. Moreover these singularities are spacelike and are of different kind from the singularities one would like to study in cosmology.

In this paper, we follow the line of AdS/CFT correspondence. We construct supersymmetric IIB backgrounds which carry nontrivial dilaton and axion profiles. The dual gauge theory is also constructed explicitly. The dual theory features a time-dependent gauge coupling and a time-dependent axion coupling and is $\mathcal{N} = 2$ supersymmetric, preserving the same amount (1/4) of supersymetries as the supergravity (SUGRA) solution. Our goal is to use the dual gauge theory to try to get a better understanding about string theory in non-static spacetime and about the properties of spacetime singularity. On this, we notice that by allowing nontrivial scalars profile, our SUGRA solution can admit not just the usual pp-singularities (which are irrelevant for our studies), but also cosmological type (to be precise, null-like) singularities. Moreover since these singularities are situated at a constant x^+, their presence can in principle be detected by quantities computed in the dual field theory. For this purpose, we compare the field theory two-point functions computed from two different methods: one computed using the bulk-boundary propagator and the other computed directly from the field theory. We find that the two don't agree. In particular the SUGRA result is sensitive to the singularity of the spacetime, while the gauge theory result does not see the singularity. That the results differ is not surprising since the SUGRA result is valid in the regime where the t'Hooft coupling is large, while the field theory result is valid when the t'Hooft coupling is small (in our case it is zero since our computation is performed at the free level). As argued by [23], a field theory computation at weak coupling should still be valuable in capturing the singularity behaviour of

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the spacetime, even if only qualitative. Therefore our field theory result suggests that the spacetime singularity as seen at the SUGRA level could be resolved by α' effects of string theory. A similar suggestion has also been reached at [23].

The paper is organized as follows. In section 2, we present our construction of the time-dependent solution in type IIB supergravity. In section 3 we present our construction of the time-dependent supersymmetric gauge theory and give justifications. In section 4, we apply the duality and try to use gauge theory to learn about the dual spacetime properties. In particular we compute the two-point functions and study their behaviour in relation to spacetime singularity structure. We also show how the finiteness of the vacuum expectation value (VEV) of the energy momentum tensor may allow us to derive the Einstein equations in the bulk. We end in section 5 with a few remarks and further discussions.

2. Time-dependent deformation of AdS background

We are interested in generalizing the original Maldacena AdS/CFT correspondence to one where the asymptotic AdS space is deformed to have nontrivial time-dependence. This will be a new class of duality different from existing studies of time-dependent processes in AdS/CFT correspondence. We will work with the extension of the AdS/CFT correspondence for the type IIB case. It should be straightforward to generalize our analysis to the 11 dimensional $AdS_4 \times S^7$ or $AdS_7 \times S^4$ cases.

In this paper, we will consider solutions where the NSNS and RR 2-form potentials vanish. Generalization to include nontrivial 2-form potentials is possible. In Einstein frame, the equations of motion for the bosonic fields are [24]

$$\nabla_M(e^{2\phi}\nabla^M\chi) = 0, \tag{2.1}$$

$$\nabla^M\nabla_M\phi + e^{2\phi}\partial^M\chi\partial_M\chi = 0, \tag{2.2}$$

$$eF_{M_1\cdots M_5} = \varepsilon_{M_1\cdots M_5}{}^{L_1\cdots L_5}F_{L_1\cdots L_5}, \tag{2.3}$$

$$R_{MN} = \frac{1}{2}\partial_M\phi\partial_N\phi + \frac{1}{2}e^{2\phi}\partial_M\chi\partial_N\chi + \frac{1}{6}F_{L_1\cdots L_4 M}F^{L_1\cdots L_4}{}_N. \tag{2.4}$$

Here $M, N = 0, 1, \cdots, 9$. e^2 is the determinant of the metric g_{MN}.

2.1 The solution

To solve (2.1) -(2.4) with non-trivial time-dependence, we will need to impose an appropriate ansatz. Since a time-dependence in the S^5 part of the metric is harder to interpret in the dual gauge theory, we will restrict ourselves in this paper to deformations only in the AdS_5 part of the metric. We start with the following ansatz ($i = 2, 3$),

$$ds^2 = \frac{R^2}{u^2}\left(-k^2(x^+)dx^+dx^- + h(u, x^-, x^+, x^i)(dx^+)^2 + (dx^i)^2 + du^2\right) + R^2 d\Omega_5^2, \tag{2.5}$$

$$F_{\mu\nu\rho\lambda\sigma} = \frac{1}{R}\varepsilon_{\mu\nu\rho\lambda\sigma}, \quad F_{abcde} = \frac{1}{R}\varepsilon_{abcde}, \text{ for the } AdS_5\text{-like and } S^5 \text{ part respectively,} \tag{2.6}$$

$$\phi = \phi(x^+), \quad \chi = \chi(x^+), \tag{2.7}$$

where the tangent space components of the 5-form is given in (2.6) and the functions k, h, ϕ, χ are defined over the whole real axis. A couple of remarks follow. 1. We have chosen not to include any x^+ dependence in front of the du^2 term as we would like to maintain the interpretation of u as the holographic energy scale, and a time dependence in it would render it difficult for such an interpretation. 2. By a redefinition of x^+, one can normalize k to 1 generically. Here we leave the possibility of an explicit x^+ dependence in k. This will allow to include singular solutions which are geodesically incomplete (see section 2.3) without having to introduce singularities in the metric. 3. Similar solutions which are asymptotically AdS were studied before The major feature which distinguishes our solution from existing solutions is that nontrivial (time-dependent) dilaton and axion configurations are turned on in our solution[1] [2]. This allows us to include solutions with spacetime singularity in the bulk.

We now consider the equations of motion. The self-duality condition (2.3) is satisfied by our ansatz. Also the equations (2.1) and (2.2) are trivially satisfied since the scalars depend only on x^+. For the Einstein equation (2.4), we note that the metric (2.5) has a modified Ricci tensor whose nonvanishing components are given by

$$R_{uu} = R_{22} = R_{33} = -\frac{4}{u^2}, \tag{2.8}$$

$$R_{+-} = \frac{2k^2}{u^2} - \frac{1}{k^2}\partial_-^2 h, \qquad R_{+I} = -\frac{1}{k^2}\partial_-\partial_I h, \quad I = 2, 3, u, \tag{2.9}$$

$$R_{++} = -\frac{4h}{u^2} + \frac{3\partial_u h}{2u} + \frac{2}{k^4}h\partial_-^2 h - \frac{1}{2}(\partial_u^2 h + \partial_2^2 h + \partial_3^2 h). \tag{2.10}$$

It is easy to see that the Einstein equation (2.4) can be satisfied if h is of the form

$$h = h_0 + h_- x^- + \sum_i \left(k_i x^i + h_{ij}\, x^i x^j\right) + l_u u^2 + (l_0 + \sum_i l_i\, x^i) u^4, \tag{2.11}$$

where all the coefficients $h_0, \cdots, l_3$ above are functions of x^+; and they satisfy

$$\frac{1}{2}(\phi')^2 + \frac{1}{2}e^{2\phi}(\chi')^2 = -h_{22} - h_{33} + 2l_u, \tag{2.12}$$

which follows from the (++)-component of the Einstein equation. In [13], a nondilatonic background was considered and solution with nonzero $h_{22} = h_{33} = l_u/2$ and l_0 was constructed. As is usual in the AdS/CFT correspondence,[3] adding u-dependent parts in h that do not change the value of h on the boundary $u = 0$ corresponds to the same boundary theory in an excited state. It has been argued in [13] that this background is dual to a field theory with a constant lightcone momentum density. In this paper we consider another type of deformation which changes the boundary metric nontrivially. For this purpose, it is sufficient to ignore the u-dependence in (2.11) and consider the ansatz

$$h = h_0 + h_- x^- + \sum_i \left(k_i x^i + h_{ij} x^i x^j\right). \tag{2.13}$$

[1] In [26], the dilaton is turned on for other branes but not the D3-brane which is the one related to our solutions

[2] We thank Arkady Tseytlin for pointing out to us that our solution can be related to those in [27] by a chain of duality relations and with dilaton and axion fields turned on.

[3] We recall that in the AdS/CFT correspondence, solutions in the bulk of AdS are matched with either physical states or background deformations of the boundary theory according to their radial dependence.

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It is now straightforward to show that by performing a coordinate transformation of the form

$$dx^+ = f(\tilde{x}^+)d\tilde{x}^+, \quad x^i = \tilde{x}^i + a^i(\tilde{x}^+), \quad u = \tilde{u}, \tag{2.14}$$

$$\tilde{x}^- = f(\tilde{x}^+)\left(x^- + b_i(\tilde{x}^+)\tilde{x}^i\right) + \tilde{\alpha}(\tilde{x}^+), \tag{2.15}$$

one can turn h_0, h_-, k_i to zero. Therefore it is sufficient to consider

$$h = h_{ij}(x^+)x^i x^j \tag{2.16}$$

for the metric and the Einstein equation reads

$$\frac{1}{2}(\phi')^2 + \frac{1}{2}e^{2\phi}(\chi')^2 = -h_{22} - h_{33}. \tag{2.17}$$

Since h is independent of x^-, our metric admits a null Killing vector $\xi = \partial_-$ and corresponds to a pp-wave in AdS. Time-dependent solutions with a null Killing vector has also been constructed in string theory, see for example [28] for some recent discussions.

The metric (2.5) is written in the "Brinkman form", one can bring it to the"Rosen form"

$$ds^2 = \frac{R^2}{u^2}\left(-k^2(y^+)dy^+dy^- + \sum_{ij} a_{ij}(y^+)dy^i dy^j + du^2\right) + R^2 d\Omega_5^2 \tag{2.18}$$

by performing a coordinate transformation. In this form, the metric depends only on the coordinate y^+. Unlike x^+ above, y^+ is null. The coordinate transformation is complicated in general. However it simplifies in the particular case of $h_{23} = 0$, which is equivalent to $a_{23} = 0$. Writing $a_{ij} = M_i^2\delta_{ij}$, the two coordinate systems are then related by the transformation

$$x^+ = y^+, \quad x^i = M_i(x^+)y^i, \quad x^- = y^- + \sum_i \frac{1}{k^2}M_i' M_i (y^i)^2 \tag{2.19}$$

and the functions h_i are related to M_i by

$$h_i = \frac{M_i''}{M_i} - \frac{2k'M_i'}{kM_i}, \quad i = 2, 3. \tag{2.20}$$

The Ricci curvature for the modified AdS part $(M, N = +, -, 2, 3, u)$ is given by,

$$R_{MN} = -\frac{4}{u^2}g_{MN} + R_{++}\delta_{M+}\delta_{N+}, \tag{2.21}$$

where R_{++} is given by an expression involving a_{ij} and their derivatives up to the second order. The expression is a little more complicated and we don't record it here. Again the $(++)$-component of the Einstein equation gives $\frac{1}{2}(\phi')^2 + \frac{1}{2}e^{2\phi}(\chi')^2 = R_{++}$ similar to (2.17), and presents a constraint among a_{ij}, ϕ and χ. For the particular case of $h_{23} = a_{23} = 0$, we have

$$R_{++} = -\sum_{i=2,3}\left(\frac{M_i''}{M_i} - \frac{2k'M_i'}{kM_i}\right). \tag{2.22}$$

We note that our supergravity solution is invariant under a scaling transformation of the following form:

$$u \to \lambda u, \quad x^+ \to x^+, \quad x^- \to \lambda^2 x^-, \quad x^i \to \lambda x^i. \tag{2.23}$$

This scaling symmetry will play a significant role in the dual gauge theory.

Before we move on, we remark that the above construction can be obtained from a suitable near horizon limit of D3-brane solutions. Consider an ansatz of the following form

$$ds^2 = H^{-1/2}\big(-k^2(x^+)dx^+dx^- + h(r, x^+, x^i)(dx^+)^2 + (dx^i)^2\big) + H^{1/2}(dr^2 + r^2 d\Omega_5^2), \tag{2.24}$$

$$F_5 = G_5 + *G_5, \quad \text{where} \quad G_5 = \frac{k^2}{4} d(H^{-1}) \wedge dx^+ \wedge dx^- \wedge dx^2 \wedge dx^3, \tag{2.25}$$

$$\phi = \phi(x^+), \quad \chi = \chi(x^+), \tag{2.26}$$

where

$$H = 1 + \frac{R^4}{r^4}, \quad \text{for arbitrary constant } R \tag{2.27}$$

One can check that the type IIB SUGRA equations of motion are satisfied if

$$-\frac{1}{2}H^{-1}\left(\partial_r^2 h + \frac{5}{r}\partial_r h\right) - \frac{1}{2}(\partial_2^2 + \partial_3^2)h = \frac{1}{2}(\phi')^2 + \frac{1}{2}e^{2\phi}(\chi')^2. \tag{2.28}$$

This can be satisfied if h takes the form (2.16) and obeys (2.17). Our solution above for pp-wave in AdS can be obtained from this solution by taking a near horizon limit $r \to 0$ and identify $u = R^2/r$. The solution (2.24)-(2.26) describes a stack of D3-branes with a pp-wave on it.

2.2 Supersymmetry

We now show that our solution preserves 1/4 of the IIB supersymmetry [24]. The preserved supersymmetry is determined by the vanishing of the variations of the spinor $\delta\lambda$ and the gravitino $\delta\psi_M$. For our solutions where the RR and NSNS 2-form potentials are zero, we have

$$0 = \delta\lambda = i\gamma^M \epsilon^* P_M, \qquad 0 = \delta\psi_M = \left(D_M - \frac{i}{2}Q_M\right)\epsilon + \frac{i}{480}F_{P1\cdots P_5}\gamma^{P_1\cdots P_5}\gamma_M\epsilon. \tag{2.29}$$

Here P_M and Q_M are currents defined by $P_M = -\varepsilon_{\alpha\beta}V_+^\alpha \partial_M V_+^\beta$, $Q_M = -i\varepsilon_{\alpha\beta}V_-^\alpha \partial_M V_+^\beta$, where $V_\pm^{\alpha=1,2}$ is a 2×2 matrix of the scalar fields which satisfies $\varepsilon V_-^\alpha V_+^\beta = 1$ and $V_-^\alpha V_+^\beta - V_+^\alpha V_-^\beta = \varepsilon^{\alpha\beta}$. In (2.29), $D_M = \partial_M + \frac{1}{4}\omega_M{}^{AB}\Gamma_{AB}$ is the covariant derivative, $\omega_M{}^{AB}$ is the spin connection matrix. Γ_A and $\gamma_M = E_M{}^A\,\Gamma_A$ are the Γ matrices in the tangent space and coordinate basis respectively: $\{\gamma_M, \gamma_N\} = 2g_{MN}, \{\Gamma_A, \Gamma_B\} = 2\eta_{AB}$.

To verify the supersymmetry, it is convenient to perform a change of coordinate $u = e^{-r}$ and use the orthonormal basis

$$E^r = dr, \qquad E^+ = e^r dx^+, \qquad E^- = e^r(k^2 dx^- - h dx^+), \qquad E^i = e^r dx^i, \quad i = 2, 3. \tag{2.30}$$

We are choosing the units so that $R = 1$ for simplicity. The metric takes the form

$$ds^2 = \eta_{AB} E^A E^B, \quad \text{where} \quad \eta_{+-} = \eta_{-+} = -1/2, \quad \eta_{22} = \eta_{33} = \eta_{rr} = 1. \tag{2.31}$$

The nonvanishing components of the spin connection ω^{AB} ($dE^A + \omega^A{}_B E^B = 0$) are given by

$$\begin{aligned} &\omega^{+r} = -\omega^{r+} = E^+, \quad \omega^{-r} = -\omega^{r-} = E^- - \partial_r h E^+, \quad \omega^{ir} = -\omega^{ri} = E^i, \\ &\omega^{i-} = -\omega^{-i} = e^{-r} \partial_i h E^+, \quad \omega^{+-} = -\omega^{-+} = \frac{2e^{-r}}{k^2} (\partial_- h + 2kk') E^+. \end{aligned} \tag{2.32}$$

Here we have given the results for the general case of the metric (2.5) where h is allowed to depend on r and x^-, although this is not the case for the solutions we construct in this paper.

Now we examine the supersymmetry conditions (2.29) for our solution. Since the scalars in our solution are functions of x^+, the only nonvanishing components of P_M, Q_M are P_+, Q_+. The condition $\delta\lambda = 0$ reads

$$\gamma^+ \epsilon = \gamma_- \epsilon = 0, \qquad \text{or, equivalently} \qquad \Gamma^+ \epsilon = \Gamma_- \epsilon = 0. \tag{2.33}$$

Due to this condition, the modification of k and h in the spin connection (2.32) never appears since a Γ_- is always attached and will give zero upon hitting ϵ. As a result, the equation (2.29) reads the same as in the undeformed $AdS_5 \times S^5$ case. The AdS_5 part of it gives

$$\partial_- \epsilon = 0, \qquad \left(\partial_+ - \frac{i}{2} Q_+ - \frac{e^r}{2} \Gamma_+ (1 - \Gamma_r) \right) \epsilon = 0, \tag{2.34}$$

$$\left(\partial_r - \frac{1}{2} \Gamma_r \right) \epsilon = 0, \quad \left(\partial_i - \frac{e^r}{2} \Gamma_i (1 - \Gamma_r) \right) \epsilon = 0. \tag{2.35}$$

These are satisfied if

$$\epsilon = e^{r/2} \epsilon_0^+, \quad \text{where} \quad \Gamma^r \epsilon_0^\pm = \pm \epsilon_0^\pm \tag{2.36}$$

for the constant spinors $\epsilon_0^\pm$. This is just the usual Poincare supersymmetries of AdS, but with the extra condition (2.33) imposed. It is easy to see that the "AdS-supersymmetry" $\epsilon = (e^{r/2} + e^{-r/2} \not{x}) \epsilon_0^-$ for the standard $AdS_5 \times S^5$ is broken since it is not compatible with (2.33). Thus our solution preserves 8 supersymmetries of the form $\epsilon = e^{r/2} \epsilon_0^+$ satisfying $\Gamma^+ \epsilon = (1 - \Gamma^r) \epsilon = 0$.

In the above we have assumed that, given that the metric is nontrivially dependent on x^+, the scalars also depend on x^+ nontrivially. However there is an interesting exception. Consider the case with

$$h_{22} = -h_{33}. \tag{2.37}$$

In this case, the dilaton and axion are constant. Although the currents are now $P_M = Q_M = 0$ and there is no need to impose $\gamma^+ \epsilon = 0$ in order for $\delta\lambda = 0$, this condition is needed in order to solve $\delta\psi_M = 0$. The solution is 1/4 BPS as before.

The next simplest example of our solutions is a linear dilaton background

$$\phi = \sqrt{-2(h_{22} + h_{33})}\, x^+, \qquad \chi = 0, \tag{2.38}$$

where $h_{22} + h_{33}$ is a constant.

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2.3 Singularity: geodesic incompleteness

An interesting feature of our supergravity solutions is that all gauge invariant quantities constructed out of the curvature tensor and the metric are regular. This does not mean that there is no singularity of any kind in our solutions. For example, a divergence in the dilaton or axion is still a singularity. This happens whenever the component R_{++} of the Ricci tensor has a singularity according to (2.17). In general a spacetime is singular if it is geodesically incomplete and cannot be embedded in a larger spacetime [29]. We will now demonstrate that our solutions include situation where there can be a singularity in the bulk. This is in contrast to those solutions constructed in e.g. [13, 21, 22]. For the following analysis, it is more convenient to consider the Rosen form (2.18) of the metric.

We first claim that if we choose $k = 1$ by redefining the coordinate y^+, then a singularity in R_{++} also signifies geodesic incompleteness. The reason is that the curve defined by

$$y^+ = \lambda, \qquad y^M = \text{constant} \quad \forall \quad M \neq + \tag{2.39}$$

is a geodesic with the affine parameter $\lambda \in \mathbb{R}$ [4]. On this geodesic,

$$R_{MN} \frac{dy^M}{d\lambda} \frac{dy^N}{d\lambda} = R_{++} \tag{2.40}$$

is invariant under general coordinate transformations. Thus a divergence of R_{++} implies that the geodesic has to be terminated there and the geodesic is incomplete. A simple example is:

$$\phi = \pm 2\sqrt{\alpha(1-\alpha)} \log(y^+), \quad \chi = 0, \quad M_2 = M_3 = (y^+)^\alpha, \quad a_{23} = 0, \quad k = 1. \tag{2.41}$$

Another example is

$$\phi = \phi_0, \quad \chi = \pm 2\sqrt{\alpha(1-\alpha)} e^{-\phi_0} \log(y^+), \quad M_2 = M_3 = (y^+)^\alpha, \quad a_{23} = 0, \quad k = 1. \tag{2.42}$$

where $y^+ > 0$ and $0 < \alpha < 1$ in both cases and ϕ_0 is a constant. From (2.22), we have a singularity at $y^+ = 0$

$$R_{++} = \frac{2\alpha(1-\alpha)}{(y^+)^2}. \tag{2.43}$$

Note that there is a vanishing scale factor resembling the big bang.

Via a coordinate transformation of y^+, which results in a nontrivial function k in the metric, it is possible to push the singularity to the coordinate infinity, so that all the fields in our solution appear to be smooth functions. For example, the above singularity at $y^+ = 0$ may be pushed to the infinity $y^+_{\text{new}} \to -\infty$ in terms of the new coordinate $y^+_{\text{new}} = \log y^+$. Simultaneously we have $k^2 = 1 \to k^2 = e^{y^+_{\text{new}}}$. This is why we claimed earlier that by allowing nontrivial k to appear in the g_{+-} component of our metric, one has the possibility to include singular spacetime which is geodesically incomplete even if all the fields (the dilaton, axion and all functions defining the metric) are seemingly regular functions.

[4]The lines $y^- = \lambda, y^M =$ constant for all $M \neq -$, also form a family of null geodesics. Since our metric is independent of y^-, it is not very interesting to follow the flow of this kind of geodesics.

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In general, let us consider the geodesic equation for the case of a nontrivial k

$$\frac{d^2y^+}{d\lambda^2} + \Gamma^+_{++}\frac{dy^+}{d\lambda}\frac{dy^+}{d\lambda} = 0, \quad \text{where } \Gamma^+_{++} = 2k'/k. \tag{2.44}$$

Up to a constant normalization of the affine parameter λ, this gives

$$\frac{dy^+}{d\lambda} = 1/k^2. \tag{2.45}$$

This implies a monotonic relation between λ and y^+ and could demand λ to be terminated at a finite value. Moreover the gauge invariant curvature defined along the geodesic is

$$R_{\lambda\lambda}(\lambda) \equiv R_{\mu\nu}\frac{dy^\mu}{d\lambda}\frac{dy^\nu}{d\lambda} = \frac{1}{k^4}R_{++}\Big|_{y^+=y^+(\lambda)}. \tag{2.46}$$

Even if all scalar profiles are smooth and so R_{++} is nonsingular for all $y^+ \in \mathbb{R}$, $R_{\lambda\lambda}$ can be singular at finite λ if k approaches to zero somewhere. In this case the spacetime is geodesic incomplete and has a curvature singularity. On the other hand, if we choose the coordinates with $k = 1$, and all the fields are smooth functions, the solution is free from any singularity in the bulk. (At the horizon $u = \infty$, there still exists pp-singularity as discussed above.)

In the above we have considered the metric in the Einstein frame. The string metric g^s_{MN} differs from the Einstein metric g_{MN} by a factor depending on the dilaton field

$$g^{(s)}_{MN} = e^{\phi/2}g_{MN}. \tag{2.47}$$

Since the dilaton is in general a nontrivial function, it may happen that a singularity in the Einstein frame disappears in the string frame. This is a very interesting situation because the string is now coupled to a nonsingular metric and if there is no other singularity, e.g. in the dilaton, then the theory should be well defined. Taking advantage of this, a nonperturbative matrix string formulation has been proposed recently to describe a null cosmological singularity in the Einstein frame while in the string frame it is a flat metric with a linear dilaton background [18].

Interestingly, our solutions also include geometry of this kind. An explicit example is that the Einstein frame metric is given by (2.18) with $a_{ij} = \delta_{ij}M^2$ and

$$k^2 = M^2 = e^{-\phi/2} = \left(\frac{y^+}{y^+_0}\right)^{2/3}. \tag{2.48}$$

The affine parameter is $\lambda = 3(y^+)^{5/3}/5$ and the gauge invariant curvature is $R_{\lambda\lambda} = \frac{8}{9}(\frac{y^+}{y^+_0})^{-\frac{10}{3}}$. The metric is singular at $y^+ = 0$ and corresponds to the $\lambda = 0$. The corresponding string metric is just the undeformed $AdS_5 \times S^5$ and is regular. Other examples are also possible.

In conclusion we have shown that our class of solutions is general enough to include both regular and singular spacetime. Moreover it includes spacetimes that are singular in the Einstein frame but regular in the string frame. This kind of spacetime is of interest for the studies of big bang cosmology. In the next section, we give a candidate supersymmetric gauge theory that we propose to be dual to our supergravity solution in general. We emphasis that this includes also the singular case.

3. Time-dependent supersymmetric Yang-Mills theory

Due to the form of our SUGRA metric, it is clear that the boundary manifold is equipped with a natural conformal structure but not a natural metric. According to [30], it is plausible that there is a correspondence between the conformal theory on the boundary and quantum gravity in the bulk. However the precise form of the boundary theory was not given. In this section, we will construct the dual theory directly. The dual theory we construct is a time-dependent Yang-Mills theory with precisely the same amount of functional dependence on x^+ and the same amount of supersymmetries as our supergravity solution. We remark that the work of [31] considered super Yang-Mills theory on a generic curved spacetime with Killing spinors. Here we have a specific choice of the metric but our action is more general in that we will allow for a time-dependent Yang-Mills coupling and also we will introduce a non-topological axion term to the action. Both are necessary since our SUGRA solution has a nontrivial dilaton and axion background.

3.1 Construction

The $\mathcal{N} = 4$ super Yang-Mills theory can be understood as the dimensional reduction of the 10 dimensional super Yang-Mills theory. Its action is

$$S = \frac{1}{g_{YM}^2} \int d^4x \mathrm{Tr} \left(-\frac{1}{4} F_{MN}F^{MN} - \frac{1}{2}\bar{\Psi}\Gamma^M [D_M, \Psi] \right), \tag{3.1}$$

where Ψ is a 10D Majorana-Weyl spinor and

$$F_{MN} = i[D_M, D_N], \qquad D_M = \begin{cases} \partial_\mu - iA_\mu & (\mu = 0, 1, 2, 3), \\ -iA_a & (a = 4, \cdots, 9). \end{cases} \tag{3.2}$$

The action is invariant under the supersymmetry (SUSY) transformation

$$\delta A_M = \frac{1}{2}\bar{\epsilon}\Gamma_M \Psi, \qquad \delta\Psi = -\frac{1}{4}F_{MN}\Gamma^{MN}\epsilon \tag{3.3}$$

for both $\epsilon = \eta$ (Poincare SUSY) and $\epsilon = x^\mu \Gamma_\mu \eta$ (conformal SUSY), where η is an arbitrary constant Majorana-Weyl spinor, and $\Gamma^{MN} = \frac{1}{2}[\Gamma^M, \Gamma^N]$. We use the convention that $\eta_{MN} = \mathrm{diag}(-1, 1, \cdots, 1)$.

For compassion with our supergravity solution, we can also go to the lightcone coordinate. Define

$$\Gamma_\pm = \frac{1}{2}(\Gamma_0 \pm \Gamma_1) \tag{3.4}$$

so that

$$\{\Gamma_+, \Gamma_-\} = -1. \tag{3.5}$$

The Minkowski metric has $\eta_{+-} = \eta_{-+} = -1/2$. Using (3.5), we note that a generic fermion Ψ can always be decomposed as

$$\Psi = \Psi_{+-} + \Psi_{-+}, \tag{3.6}$$

where

$$\Psi_{+-} = -\Gamma_+\Gamma_-\Psi, \qquad \Psi_{-+} = -\Gamma_-\Gamma_+\Psi. \tag{3.7}$$

In view of the supersymmetry preserved by our supergravity solution, we look for supersymmetric Yang-Mills theories (SYM) preserving the supersymmetry for ϵ being a constant 10D Majorana-Weyl spinor satisfying

$$\Gamma_-\epsilon = 0. \tag{3.8}$$

In this case, the SUSY transformation parameter satisfies

$$\Gamma_-\Gamma_+\epsilon = -\epsilon. \tag{3.9}$$

Let us start by rewriting the original SYM action (3.1) for a curved base space:

$$S_0 = \int d^4x(\mathcal{L}_{0B} + \mathcal{L}_{0F}), \tag{3.10}$$

where

$$\mathcal{L}_{0B} = f_B \mathrm{Tr}\left[-\frac{1}{4}\tilde{g}^{MM'}\tilde{g}^{NN'}F_{MN}F_{M'N'}\right], \tag{3.11}$$

$$\mathcal{L}_{0F} = f_F \mathrm{Tr}\left[-\frac{1}{2}\bar{\Psi}\gamma^M[D_M,\Psi]\right]. \tag{3.12}$$

Since the base space is curved, the covariant derivative D_M also includes the spin connection when it acts on Ψ. The functions f_B and f_F are assumed to be functions of x^+ only. We will match f_B with the factor $\frac{\sqrt{-g}}{g_{YM}^2}$, where both the measure $\sqrt{-g}$ and Yang-Mills coupling g_{YM} are time-dependent functions. Naturally we also deform the SUSY transformation (3.3) to include two functions $f_A(x^+)$ and $f_\Psi(x^+)$

$$\delta A_M = \frac{1}{2}f_A\bar{\epsilon}\gamma_M\Psi, \qquad \delta\Psi = -\frac{1}{4}f_\Psi F_{MN}\gamma^{MN}\epsilon \tag{3.13}$$

For the parameter ϵ satisfying (3.8), these can be written in more detail as

$$\delta A_- = 0, \tag{3.14}$$

$$\delta A_+ = \frac{1}{2}f_A\bar{\epsilon}\gamma_+\Psi_{-+}, \tag{3.15}$$

$$\delta A_m = \frac{1}{2}f_A\bar{\epsilon}\gamma_m\Psi_{+-} \quad \text{for} \quad m = 2,3,\cdots,9, \tag{3.16}$$

$$\delta\Psi_{+-} = -f_\Psi F_{-m}\gamma^m\gamma_+\epsilon, \tag{3.17}$$

$$\delta\Psi_{-+} = f_\Psi\left(F_{-+} - \frac{1}{4}\sum_{m,n=2}^{9}F_{mn}\gamma^{mn}\right)\epsilon. \tag{3.18}$$

We will use indices $M,N = +,-,2,3,\ldots,9$, and indices $m,n = 2,3,\ldots,9$. Below we will also use $i,j = 2,3$ and $a,b = 4,5,\ldots,9$.

Choose the metric of the base space to be of the form

$$ds^2 = \tilde{g}_{\mu\nu}dx^\mu dx^\nu = -\tilde{k}^2dx^+dx^- + \tilde{h}dx^+dx^+ + dx^idx^i. \tag{3.19}$$

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We remark that, here we use $\tilde{g}_{\mu\nu}, \tilde{k}, \tilde{h}$ to denote quantities in the Yang-Mills theory, in order to distinguish them from the SUGRA quantities: $g_{\mu\nu}, k, h$. For the vielbein, we take

$$E^- = \tilde{k}^2 dx^- - \tilde{h} dx^+, \qquad E^M = dx^M \quad \text{for} \quad M \neq -, \tag{3.20}$$

where we have extended the definition of the vielbein to the indices $a = 4, \cdots, 9$, so that we can define

$$\gamma_M = E_M{}^A \Gamma_A. \tag{3.21}$$

We have $\{\gamma_M, \gamma_N\} = 2\tilde{g}_{MN}$. A different choice of vielbein is possible and corresponds to a different choice of the spin connection. For our choice, the spin connection can be easily read off from (2.32) by setting $r = dr = 0$.

One can check that the variation of the action (3.10) for a generic base space is

$$\begin{aligned} \delta S_0 = \int d^4x \ \mathrm{Tr} \Big[-\frac{1}{2}\bar{\epsilon}\Big(\frac{1}{4} f_\Psi \gamma_{MN} [D_K, f_F \gamma^K]\Psi + f_B [D_M, f_A \gamma_N]\Psi \\ +(f_F f_\Psi - f_B f_A)\gamma_M [D_N, \Psi]\Big) F^{MN}\Big]. \end{aligned} \tag{3.22}$$

For our choice of the vielbein (3.20), both terms vanish for ϵ satisfying (3.8) if

$$f_B f_A - f_F f_\Psi = 0, \qquad f_B f_A' - f_F' f_\Psi = 0. \tag{3.23}$$

When the coupling is constant [31], we can choose $f_B = f_F = \sqrt{-g}/g_{YM}^2$ and $f_A = f_\Psi = \sqrt{-g}$. In our case the coupling g_{YM} is not a constant, by convention we identify f_B with $\sqrt{-g}/g_{YM}^2$. (This is how one defines g_{YM}^2). Note that one can scale Ψ by an arbitrary function of x^+. In particular we can scale Ψ such that f_F is equal to f_B. This implies that f_A equals f_Ψ and so the solution is

$$f_B = f_F = f_A = f_\Psi = \frac{\sqrt{-g}}{g_{YM}^2} \tag{3.24}$$

up to scaling f_A and f_Ψ by a constant, which is equivalent to scaling ϵ. We see that the super Yang-Mills theory can have a generic coupling function g_{YM} depending on x^+.

There is an additional term invariant under the same SUSY transformation

$$\mathcal{L}_\chi = \tilde{\chi}(x^+)\mathrm{Tr}\left(\frac{1}{4}\epsilon^{\mu\nu\rho\sigma} F_{\mu\nu} F_{\rho\sigma} + \frac{1}{2}\frac{f_A}{f_\Psi}\bar{\Psi}_{+-}\Gamma^2\Gamma^3\Gamma_-\Psi_{+-}\right), \tag{3.25}$$

where $\epsilon^{\mu\nu\alpha\beta}$ is the totally antisymmetrized tensor with $\epsilon^{+-23} = 1$. It is not hard to show that under the variations (3.14)–(3.18), $\mathcal{L}_2$ is invariant for an arbitrary function $\tilde{\chi}(x^+)$. In conclusion, a super Yang-Mills Lagrangian invariant under the transformation (3.14)–(3.18) is

$$\mathcal{L} = \mathcal{L}_{0B} + \mathcal{L}_{0F} + \mathcal{L}_\chi. \tag{3.26}$$

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Explicitly, the total action is

$$
\begin{aligned}
S = &\int d^4x \, \frac{\sqrt{-g}}{g_{YM}^2} \mathrm{Tr}\Bigg[2\tilde{k}^{-4} F_{+-}^2 + 2\tilde{k}^{-2} F_{+i} F_{-i} - \frac{1}{2} F_{23}^2 \\
&+2\tilde{k}^{-2} [D_-, X_a][D_+, X_a] - \frac{1}{2}[D_i, X_a]^2 + \frac{1}{4}[X_a, X_b]^2 \\
&-4\tilde{k}^{-4}\tilde{h}(x^+, x^2, x^3)\left(\frac{1}{2}F_{-i}^2 + \frac{1}{2}[D_-, X_a]^2\right)\Bigg] \\
&+ \int d^4x \frac{\sqrt{-g}}{g_{YM}^2} \mathrm{Tr}\Bigg[\bar{\Psi}_{-+}\Gamma_+[D_-, \Psi_{-+}] + \bar{\Psi}_{+-}\tilde{k}^2\Gamma_-[D_+, \Psi_{+-}] - \frac{1}{2}\bar{\Psi}\Gamma^i[D_i, \Psi] + \frac{i}{2}\bar{\Psi}\Gamma^a[X_a, \Psi] \\
&+\tilde{k}^{-4}\tilde{h}(x^+, x^2, x^3)\bar{\Psi}_{+-}\tilde{k}^2\Gamma_-[D_-, \Psi_{+-}]\Bigg] \\
&+ \int d^4x \ \mathrm{Tr}\left[\tilde{\chi}(x^+)\left(\frac{1}{4}\epsilon^{\mu\nu\rho\sigma}F_{\mu\nu}F_{\rho\sigma} + \frac{1}{2}\bar{\Psi}_{+-}\Gamma^2\Gamma^3\Gamma_-\Psi_{+-}\right)\right].
\end{aligned} \tag{3.27}
$$

In the above, we have denoted A_a as X_a and $i = 2, 3$, $a = 4, \cdots, 9$. This action is invariant under the 8 supersymmetries of (3.14) - (3.18). One can verify that the conformal supersymmetry are all broken. Thus our theory preserves 8 supersymmetries.

The above is for general $\tilde{h}$. We note that if the function $\tilde{h}$ is bilinear in x^2, x^3:

$$
\tilde{h}(x^+, x^2, x^3) = \tilde{h}_{ij}(x^+)x^i x^j, \tag{3.28}
$$

then our action (3.27) enjoys the scaling symmetry:

$$
\begin{aligned}
&x^+ \to x^+, \qquad x^- \to \lambda^2 x^-, \qquad x^i \to \lambda x^i, && (3.29) \\
&\Gamma_+ \to \lambda\Gamma_+, \qquad \Gamma_- \to \lambda^{-1}\Gamma_-, \qquad \Gamma_i \to \Gamma_i, \qquad \Gamma_a \to \Gamma_a, && (3.30) \\
&A_+ \to A_+, \quad A_- \to \lambda^{-2}A_-, \quad A_i \to \lambda^{-1}A_i, \quad X_a \to \lambda^{-1}X_a, \quad \Psi \to \lambda^{-3/2}\Psi,
\end{aligned}
$$

since $\tilde{h}$ scales like

$$
\tilde{h}(x^+, x^2, x^3) \to \lambda^2 \tilde{h}(x^+, x^2, x^3) \tag{3.31}
$$

in this case. This symmetry is also a symmetry of the original AdS/CFT background since it can be viewed as a combination of the usual scaling symmetry

$$
x^\mu \to \lambda x^\mu \tag{3.32}
$$

and the Lorentz boost in the x^1 direction

$$
x^+ \to \lambda^{-1}x^+, \quad x^- \to \lambda x^-, \quad x^i \to x^i. \tag{3.33}
$$

However the full scaling symmetry (3.32) is broken in our case and only the partial scaling symmetry is respected by our solution.

3.2 Born-Infeld analysis

In this section, we propose, and give further justification, that the time-dependent SYM theory we constructed in section 3.1 is dual to the the string theory based on the time-dependent IIB background we constructed in section 2.

Our proposal is that the time-dependent SYM theory (3.10) with Yang-Mills coupling given by

$$g_{YM}^2 = g_s \equiv e^{\phi} \tag{3.34}$$

and $\tilde{h}$ given by (3.28) provides a dual description of the string theory based on the time-dependent IIB background (2.5)-(2.7) with h given by (2.16). Moreover we propose the following identification:

$$h = \tilde{h}, \qquad \chi = \tilde{\chi}, \qquad k = \tilde{k}. \tag{3.35}$$

We remind the reader again that the left hand side are SUGRA quantities and the right hand side are SYM quantities.

Let us now explain and justify our proposal. Consider a single D3-brane in our supergravity background. The D3-brane action is given by the DBI action plus a coupling to the RR gauge fields

$$S = -\mu_3 \int d^4x e^{-\phi} \left[-\det\left(G_{\mu\nu} + \mathcal{F}_{\mu\nu}\right)\right]^{1/2} + \int C \wedge e^{\mathcal{F}}, \tag{3.36}$$

where

$$\mathcal{F}_{\mu\nu} \equiv B_{\mu\nu} + 2\pi\alpha' F_{\mu\nu}, \tag{3.37}$$

$$C = C^{(0)} + C^{(2)} + \cdots, \tag{3.38}$$

are the RR gauge fields, and

$$G_{\mu\nu} = \frac{\partial X^M}{\partial x^\mu}\frac{\partial X^N}{\partial x^\nu} g^{(s)}_{MN}, \quad B_{\mu\nu} = \frac{\partial X^M}{\partial x^\mu}\frac{\partial X^N}{\partial x^\nu} B_{MN} \tag{3.39}$$

are the pull back to D3-brane worldvolume of the spacetime metric in the string frame and the NSNS B-field.

By performing a weak field expansion, we have

$$S_{DBI} = -\mu_3 (2\pi\alpha')^2 \int d^4x e^{-\phi}\sqrt{-\det(G_{\mu\nu})}\left(-\frac{1}{4}F_{\mu\mu'}F_{\nu\nu'}G^{\mu\nu}G^{\mu'\nu'} + \cdots\right), \tag{3.40}$$

where $\cdots$ denotes higher order terms in FG^{-1}. Now let us consider a D3-brane placed at $u = u_0$ and extends in the $+, -, 2, 3$ directions. Take a static gauge

$$x^\mu = X^\mu, \quad \mu = +, -, 2, 3. \tag{3.41}$$

We have

$$G_{\mu\nu} = \frac{R^2}{u_0^2} e^{\phi/2} \hat{g}_{\mu\nu}, \tag{3.42}$$

where

$$\hat{g}_{\mu\nu} = \begin{pmatrix} h & -k^2/2 & 0 & 0 \\ -k^2/2 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}. \tag{3.43}$$

Substituting into (3.40), we have

$$S_{DBI} = -\mu_3(2\pi\alpha')^2 \int d^4x \left[-\frac{1}{4}\sqrt{\hat{g}} e^{-\phi} F_{\mu\mu'} F_{\nu\nu'} \hat{g}^{\mu\nu} \hat{g}^{\mu'\nu'} \right] + \cdots . \tag{3.44}$$

This is precisely the form of the SYM action (3.10) by identifying the string coupling with the Yang-Mills coupling in the usual way

$$e^{\phi} = g_{YM}^2, \tag{3.45}$$

and identifying $\tilde{g}_{\mu\nu}$ of (3.19) with $\hat{g}_{\mu\nu}$ here. The later implies that

$$\tilde{k} = k, \quad \tilde{h} = h. \tag{3.46}$$

Moreover since in our case $B = 0$ and there is only the $C^{(0)}$ RR field, the RR coupling reduces to $\int C^{(0)} F \wedge F$ and can be identified with the SYM piece $\int \mathcal{L}_\chi$ directly. Therefore we find that the weak field expansion of the bosonic D3-brane action produces precisely our time-dependent SYM theory. We remark that the higher order terms in (3.44) vanish as we take $u_0 \to 0$.

This justifies our choice (3.34) and our identification (3.35) for the functions which appear in our supergravity solution and in our SYM Lagrangian. For N D3-branes the action is given by the nonabelian generalization of (3.44). Our action (3.27) is the supersymmetric completion of it.

For the duality to be precise, we still need to determine the radius R of the SUGRA solution in terms of gauge theory parameters. Recall that our SUGRA solution can be obtained as a near horizon limit of the SUGRA solution for a stack of D3-branes with pp-wave on it. Consider a stack of N such D3-branes. If one equates the mass and charge of the D-brane, one obtain that

$$R^4 = 16\pi N \langle g_s^{-1} \rangle^{-1} l_s^4 \tag{3.47}$$

where $\langle g_s^{-1} \rangle := \int dx^+ \tilde{k}^2 e^{-\phi} / \int dx^+ \tilde{k}^2$ is the x^+-average of the inverse of the string coupling $g_s = e^{\phi}$.

Provided that $\langle g_s^{-1} \rangle$ is well defined, we propose that the time-dependent SYM theory is dual to the quantum gravity in the bulk with R given by (3.47). For instance, $\langle g_s^{-1} \rangle$ is well defined for the example (2.42). Our proposal is supported by a number of matchings. First we see that there is a precise matching between the functional dependence on x^+ of the two theories. We will also explain in section 4.2 how the Einstein equation (4.40) is realized as a constraint in the SYM theory. Furthermore, our theories also match in their various symmetries. The SYM action (3.10) enjoys a global $SO(6)$ invariance rotating the six scalars. This is mapped to the rotational symmetry of the S^5 on the supergravity side. Supersymmetry also matches. Both theories observe 8 supersymmetries, and these unbroken supersymmetries satisfy the same chirality condition $\Gamma^+\epsilon = 0$. Moreover, as we noted above, both the supergravity solution and the SYM action observe a scaling symmetry (2.23) and (3.29).

While the matching between the weak field expansion of the DBI action with the super Yang-Mills theory only makes sense in the low energy limit, it is possible that the identification of parameters (3.34) and (3.35) between the type IIB string theory and super Yang-Mills theory could be modified by higher derivative terms in the α' expansion. The matching of parameters including higher order terms can in principle be achieved order by order by comparing the type IIB stringy corrections to the supergravity equations of motion with the quantum corrections of super Yang-Mills theory via a similar calculation as the one carried out in section 4.2 but to a higher order.

4. Holographic duality

4.1 Two-point correlation functions and singularity structures

In AdS/CFT duality, the action of fluctuations in AdS space with specified boundary conditions is matched with correlation functions of the corresponding operators. As a result, the boundary bulk propagator in AdS should agree with the corresponding two-point correlation functions.

SUGRA calculation. In this subsection, we will write down the metric (2.5) in the Rosen form (2.18)

$$ds^2 = \frac{1}{u^2}(du^2 - 2dx^+dx^- + a_{ij}(x^+)dx^i dx^j) = g_{\mu\nu}dx^\mu dx^\nu. \tag{4.1}$$

The S^5 part of the spacetime will be ignored in this section for simplicity. In our case we have $d = 4$, but for generality we leave d as a variable in the following. Consider the action of a scalar field φ

$$S = \frac{1}{2}\int du d^d x\sqrt{-g}\,(g^{MN}\partial_M\varphi\partial_N\varphi + m^2\varphi^2). \tag{4.2}$$

First we want to solve the equation of motion

$$\Box\varphi - m^2\varphi = 0. \tag{4.3}$$

One has $\Box = u^{d+1}\partial_u(u^{-d+1}\partial_u) + u^2\Delta$ where

$$\Delta := \frac{1}{\sqrt{a}}\Big[-\partial_+(\sqrt{a}\partial_-) - \partial_-(\sqrt{a}\partial_+) + \partial_i(\sqrt{a}a^{ij}\partial_j)\Big] \tag{4.4}$$

is the d dimensional d'Alembertian operator of the boundary metric and we have denoted the determinant of the matrix a_{ij} by a. Take the ansatz

$$\varphi = \tilde{\varphi}(\vec{k}, u)\psi_{\vec{k}}(\vec{x}) \tag{4.5}$$

and use the technique of separation of variables, we find

$$u^{d-1}\partial_u(u^{-d+1}\partial_u\tilde{\varphi}) - (k^2 + \frac{m^2}{u^2})\tilde{\varphi} = 0 \tag{4.6}$$

and

$$\Delta\psi_{\vec{k}} = -k^2\psi_{\vec{k}} \tag{4.7}$$

for arbitrary separation constant $k^2 \in \mathbb{R}$ [5]. The equation (4.7) has solution

$$\psi_{\vec{k}}(\vec{x}) = \frac{1}{a^{1/4}}e^{i(k_i x^i - k_- x^- - \beta(x^+))}, \tag{4.8}$$

where $\vec{k}$ denotes the set (k_i, k_-, k^2),

$$\dot{\beta} = \frac{a^{ij}(x^+)k_i k_j - k^2}{2k_-} \tag{4.9}$$

and a^{ij} is the inverse matrix of a_{ij}. We note that ψ_k's form a basis of functions of x. One can check that

$$\int d^d\vec{x}\sqrt{a}\psi_k(\vec{x})\psi_{k'}(\vec{x}) = 2k_-\delta(k_i + k'_i)\delta(k_- + k'_-)\delta(k^2 - k'^2). \tag{4.10}$$

We also have

$$\int [d^d\vec{k}]\psi_k(\vec{x})\psi^*_k(\vec{y}) = \frac{1}{\sqrt{a}}\delta(x^+ - y^+)\delta(x^- - y^-)\prod_i \delta(x^i - y^i). \tag{4.11}$$

where the measure of integration is

$$\int [d^d\vec{k}] := \int_{-\infty}^{\infty} d^{d-2}k_i \int_{-\infty}^{\infty} \frac{dk_-}{2k_-} \int_{-\infty}^{\infty} d(k^2). \tag{4.12}$$

The equation (4.6) depends on the separation constant k^2. Its most general solution which is asymptotic to $\epsilon^{2h_-}\tilde{\varphi}_0(k^2)$ is

$$\tilde{\varphi}(k^2, u) = K_\epsilon(k^2, u)\tilde{\varphi}_0(k^2), \tag{4.13}$$

where

$$K_\epsilon(k^2, u) = \frac{\tilde{\varphi}^{(-)}(k^2, u) + A(k^2)\tilde{\varphi}^{(+)}(k^2, u)}{\tilde{\varphi}^{(-)}(k^2, \epsilon) + A(k^2)\tilde{\varphi}^{(+)}(k^2, \epsilon)}\,\epsilon^{2h_-} \tag{4.14}$$

is the bulk-boundary Green function. Here $h_\pm = (d \pm 2\nu)/4$ and $\nu := \frac{1}{2}\sqrt{d^2 + 4m^2} > 0$. $\tilde{\varphi}^{(-)}$ is a non-normalizable solution which behaves as u^{2h_-} as $u \to 0$, and $\tilde{\varphi}^{(+)}$ is a normalizable solution which behaves as u^{2h_+} as $u \to 0$[6]. $A(k^2)$ is an arbitrary coefficient

[5]Note that here k^2 is simply a constant of separation. One may also introduce the inner product $k \cdot k := a_{ij}(x^+)k^i k^j - k_+ k_-$ for some momentum vector k. This object however will not be used at all in this paper.

[6]Explicitly, for $k^2 < 0$, we have

$$\tilde{\varphi}^{(\pm)}(k^2, u) \propto u^{d/2}J_{\pm\nu}(|k|u). \tag{4.15}$$

if ν is non-integral. If ν is integral, the two independent solutions are $\tilde{\varphi}^{(+)}$ in (4.15) and

$$\tilde{\varphi}^{(-)}(k^2, u) \propto u^{d/2}Y_\nu(|k|u). \tag{4.16}$$

Here $|k| = +\sqrt{|k^2|}$.

which corresponds to the freedom to specify the vacuum state of the dual field theory, which is reflected in the freedom to choose different Lorentzian propagator in the dual theory.

Expanding the scalar field in terms of this basis

$$\varphi(x,u) = \int [d^d\vec{k}]\psi_{\vec{k}}(x)\tilde{\varphi}(k^2,u), \tag{4.17}$$

and plugging it into the action, we find

$$S = -\lim_{u=\epsilon\to 0}\int [d^d\vec{k}]\Big(u^{-d+1}\tilde{\varphi}(k^2,u)\partial_u\tilde{\varphi}(k^2,u)\Big). \tag{4.18}$$

Now consider an operator $\tilde{\mathcal{O}}$ on the boundary theory which couples to the field $\tilde{\varphi}_0$ with the coupling $\int [dk]\tilde{\mathcal{O}}(\vec{k})\tilde{\varphi}_0(\vec{k})$. This gives

$$\langle \tilde{\mathcal{O}}(\vec{k})\tilde{\mathcal{O}}(\vec{k}')\rangle = \frac{\delta^2 S}{\delta\tilde{\varphi}_0(\vec{k})\delta\tilde{\varphi}_0(\vec{k}')}. \tag{4.19}$$

Substitute the general solution (4.13) in (4.18), we obtain

$$\langle \tilde{\mathcal{O}}(\vec{k})\tilde{\mathcal{O}}(\vec{k}')\rangle = -\epsilon^{-d+1}\delta(k_i+k_i')\delta(k_-+k_-')\delta(k^2-k^{2\prime})2k_-\lim_{u=\epsilon\to 0}\partial_u K(k^2,u). \tag{4.20}$$

The resulting two-point function depends strongly on the coefficient $A(k^2)$. We note that the nontriviality of the metric affects only the eigenfunction $\psi_{\vec{k}}(\vec{x})$, but the modes $\tilde{\varphi}^{(\pm)}$ as well as the bulk-boundary propagator $K_\epsilon(k^2,u)$ as given in (4.14) take exactly the same form as in the standard AdS case. Thus one can imagine deforming adibatically the metric back to the undeformed AdS metric and use the same $K_\epsilon(k^2,u)$. In that case $A(k^2)$ is chosen via analytic continuation from the Euclidean and it is

$$\lim_{u=\epsilon\to 0}\partial_u K_\epsilon(k^2,u) = \epsilon^{2\nu-1}k^{2\nu} + \cdots . \tag{4.21}$$

Here $\cdots$ denotes terms that are of sub-leading order in ϵ and terms which contain integral powers in k^2, which as usual give rise to contact terms in the correlation function and thus will be ignored. To go to the coordinate space, we consider the operators

$$\hat{\mathcal{O}}(\vec{x}) := a^{\frac{1}{4}}(x^+)\int [d^d\vec{k}]\psi_{\vec{k}}(\vec{x})\tilde{\mathcal{O}}(\vec{k}), \tag{4.22}$$

$$\hat{\varphi}_0(\vec{x}) := a^{\frac{1}{4}}(x^+)\int [d^d\vec{k}]\psi_{\vec{k}}(x)\tilde{\varphi}_0(k^2). \tag{4.23}$$

Note that, different from the usual Fourier transform, we have included an additional factor of $a^{\frac{1}{4}}(x^+)$ in front. This is a natural definition in view of the identity

$$\int [dk]\mathcal{O}(\vec{k})\tilde{\varphi}_0(\vec{k}) = \int dx\hat{\mathcal{O}}(\vec{x})\hat{\varphi}_0(\vec{x}). \tag{4.24}$$

Also our results will turn out to be simpler when expressed in terms of $\hat{\mathcal{O}}$. Using

$$\langle\hat{\mathcal{O}}(\vec{x})\hat{\mathcal{O}}(\vec{y})\rangle = a^{\frac{1}{4}}(x^+)a^{\frac{1}{4}}(y^+)\int [d\vec{k}][d\vec{k}']\psi_{\vec{k}}(\vec{x})\psi_{\vec{k}'}(\vec{x}')\langle\tilde{\mathcal{O}}(\vec{k})\tilde{\mathcal{O}}(\vec{k}')\rangle, \tag{4.25}$$

it is easy to obtain

$$\langle\hat{\mathcal{O}}(\vec{x})\hat{\mathcal{O}}(\vec{y})\rangle = \frac{c}{(B_{ij}\delta x^i\delta x^j - \delta x^-)^{\frac{d}{2}+\nu}}\frac{1}{(\det B^{ij})^{\frac{1}{2}}(\delta x^+)^{\nu+1}}, \tag{4.26}$$

where c is a constant. Here $\delta x^{i,\pm} := x^{i,\pm} - y^{i,\pm}$,

$$B^{ij} := \int_{x^+}^{y^+} dx^+ a^{ij} \tag{4.27}$$

and B_{ij} is the inverse. If a_{ij} is proportional to the unit matrix as in our example (2.41) or (2.42),

$$a_{ij} = M^2\delta_{ij}, \tag{4.28}$$

then

$$B^{ij} = \delta_{ij}(f(x^+) - f(y^+)) := \delta_{ij}\cdot\delta f, \quad \text{where} \quad f(x^+) = \int^{x^+}\frac{dz^+}{M^2(z^+)} \tag{4.29}$$

and

$$\langle\hat{\mathcal{O}}(\vec{x})\hat{\mathcal{O}}(\vec{y})\rangle = \frac{c}{((\delta x^i)^2 - \delta x^-\delta f)^{\frac{d}{2}+\nu}}\left(\frac{\delta f}{\delta x^+}\right)^{\nu+1}. \tag{4.30}$$

This expression reduces to the usual result for flat space when $M^2 = 1$. For the example (2.41) or (2.42), we have $M^2 = (x^+)^\alpha$ and

$$f(x^+) = \frac{(x^+)^{1-\alpha}}{1-\alpha}. \tag{4.31}$$

Notice that (4.30) is singular if both x^+ and y^+ approach zero simultaneously ($0 < \alpha < 1$).

We remark that in general for an operator $\mathcal{O}$ with scaling dimension Δ, i.e. under the transformation (3.29), $\mathcal{O}$ transforms as $\mathcal{O}(x') = \lambda^{-\Delta}\mathcal{O}(x)$, the most general form of the two-point function that is compatible with the symmetry of the theory is:

$$\langle\mathcal{O}(\vec{x})\mathcal{O}(\vec{y})\rangle = |\delta x^-|^\Delta\, g\Big(x^+, y^+, \frac{\delta x^i}{\sqrt{|\delta x^-|}}\Big), \tag{4.32}$$

where g is an arbitrary function. Our result (4.26) as determined by the bulk-boundary propagator approach is compatible with this for $\Delta = \frac{d}{2}+\nu$. However (4.26) is more specific than the kinematical result (4.32). It is a consequence of dynamics.

Field theory calculation. Next we compute the two-point correlation function from the field theory point of view. Consider a scalar field φ which satisfies the equation of motion

$$(\Delta - m^2)\varphi = 0. \tag{4.33}$$

It has the mode expansion

$$\varphi = \int_0^\infty \frac{dk_-}{\sqrt{2k_-}}\int_\infty^\infty dk_i\ (\psi_{\vec{k}}a_{\vec{k}} + \psi^*_{\vec{k}}a^\dagger_{\vec{k}}), \tag{4.34}$$

where $\psi_{\vec{k}}$ is given in (4.7) above and $k^2 = -m^2$. The equal time commutation relation

$$[\varphi(\vec{x}), \sqrt{a}\partial_- \varphi(\vec{y})]\Big|_{x^+=y^+} = i\delta(x^- - y^-)\delta(x^i - y^i) \tag{4.35}$$

implies that

$$[a_{\vec{k}}, a^\dagger_{\vec{k}'}] = \delta(k_- - k'_-)\delta(k_i - k'_i). \tag{4.36}$$

Using this, one can easily compute the time ordered (with respect to x^+) product $\langle T\varphi(\vec{x})$ $\varphi(\vec{y})\rangle$. We have

$$G(\vec{x}, \vec{y}) := -i\langle T\varphi(\vec{x})\varphi(\vec{y})\rangle = \int [d^d\vec{k}]\, \frac{1}{k^2 + m^2 - i\epsilon}\, \psi_{\vec{k}}(x)\psi_{\vec{k}}(y)^*. \tag{4.37}$$

Here the vacuum is choosen to be anhillated by the operators $a_{\vec{k}}$. This corresponds to the choice of the bulk-boundary Green function K_ϵ above in the SUGRA calculation. For the interests in studying singular spacetime, let us consider the example (2.42) of the metric. The Green function G can be easily be computed and the result is (for $m^2 = 0$)

$$G(\vec{x}, \vec{y}) = \frac{c}{((\delta x^i)^2 - \delta x^- \delta f)^{\frac{d}{2}-1}} \frac{1}{a^{\frac{1}{4}}(x^+) a^{\frac{1}{4}}(y^+)}, \tag{4.38}$$

The additional (factorisable) factors of a suggests one to consider the rescaled field $\hat{\varphi} := a^{\frac{1}{4}}\varphi$. Note that this is the same rescaling appearing above in the definition of the dual field operators (4.23).

Now we want to compare our SUGRA result and the field theory result. In the usual case, the form of the two-point function is fixed by the conformal symmetry. In our case, the scaling symmetry does not fix the form of the two-point function uniquely. So generally there is no reason to expect the two computations to agree. In fact for an operator $\hat{\mathcal{O}} := (\hat{\varphi})^n$, the two-point function is

$$\langle \hat{\mathcal{O}}(\vec{x})\hat{\mathcal{O}}(\vec{y})\rangle = (a^{\frac{1}{4}}(x^+) a^{\frac{1}{4}}(y^+) G(\vec{x}, \vec{y}))^n = \frac{c}{\left((\delta x^i)^2 - \delta x^- \delta f\right)^{n(\frac{d}{2}-1)}} \tag{4.39}$$

in the tree level approximation. This is to be compared with (4.30). The factor involving δf fixes $\Delta = n(\frac{d}{2} - 1)$. However due to the absence of the term $(\delta f/\delta x^+)^{\nu+1}$ in (4.39), (4.30) and (4.39) cannot agree with each other . In particular the singularity structure is different. It is remarkable that (4.38) is completely regular even when $x^+, y^+ \to 0$, while the SUGRA result (4.30) is singular. Note that there is no particle creation in either the bulk or boundary theory. This can be easily checked following similar computations as [32, 33]. Hence there is no ambiguity in the two-point functions associated with the choice of vacuum.

Our interpretation of the result is the following: The SUGRA is a low energy approximation. The singularity at $x^+ = 0$ of the spacetime as revealed by the divergence in R_{++} and by the two-point function (4.30) is just a low energy description which may be modified in the full quantum gravity by string loop and α' effects. As we proposed, the quantum theory is described in terms of the dual quantum SYM we constructed in section 3. On the SYM side, the singularity structure of the spacetime, as revealed by the two-point function,

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is indeed different. Although the field theory result is computed at weak coupling, there is reason to expect that it will be able to capture the qualitative behaviour of the spacetime singularity [23]. (If we choose the plus sign for ϕ in (41), the string coupling actually goes to zero at the singularity.) Therefore our result suggests that the singularity is resolved to some extent. Although our SYM computation is preliminary as we have done it only at the free and tree level, we believe that the picture and interpretation is basically correct. What is surprising is that it seems that α' effects, rather than string loop effects, are sufficient to smoothen the spacetime singularity. More work is needed on this issue.

4.2 Einstein equation from super Yang-Mills theory

On the supergravity side, Einstein's equation imposes a constraint (2.17) among the parameterising functions

$$\frac{1}{2}(\phi')^2 + \frac{1}{2}e^{2\phi}(\chi')^2 = -h_{22} - h_{33}. \tag{4.40}$$

The proposed duality implies that this constraint should also be imposed on the super Yang-Mills theory. But why? At the classical level, the super Yang-Mills theory is well defined regardless of the Einstein equations. However, since the duality mixes classical effects and quantum effects between the dual theories, a super Yang-Mills theory is a candidate of the dual theory only if it is well defined at the quantum level. One should demand that all correlation functions of fundamental operators are well defined, and that the scaling symmetry is anomaly-free.

More specifically, we suggest that Einstein equations are obtained from the super Yang-Mills theory by demanding the vacuum expectation value (VEV) of the energy-momentum operator $T_{\mu\nu}$ to be finite. Assuming the duality, $\langle T_{\mu\nu}\rangle$ can be computed on the supergravity side [34]

$$\begin{aligned}\langle T_{\mu\nu}\rangle = -\frac{1}{8\pi G_N}\lim_{\epsilon\to 0}\Big[&\tfrac{1}{\epsilon^2}\left(-g_{(2)\mu\nu} + g_{(0)\mu\nu}\mathrm{Tr}g_{(2)} + \tfrac{1}{2}R^{(4)}_{\mu\nu} - \tfrac{1}{4}g_{(0)\mu\nu}R^{(4)}\right) \\ &+ \log\epsilon\left(-2h_{(4)\mu\nu} - T^a_{\mu\nu}\right) + \cdots\Big],\end{aligned} \tag{4.41}$$

where we only listed the diverging part of the VEV. Let us explain the notation. First, ϵ is the infrared cutoff at $u = \epsilon$. The 4D functions $g_{(0)}$, $g_{(2)}$ and $h_{(4)}$ are expansion coefficients of the metric near the AdS boundary

$$g(x,u) = g_{(0)} + g_{(2)}u^2 + g_{(4)}u^4 + h_{(4)}u^4\log u^2 + \mathcal{O}(u^5), \tag{4.42}$$

while $g(x,u)$ is the 4D part of the full metric of the 5D warped metric

$$ds^2 = \frac{R^2}{u^2}(du^2 + g_{\mu\nu}dx^\mu dx^\nu). \tag{4.43}$$

$R^{(4)}_{\mu\nu}$ and $R^{(4)}$ are the Ricci tensor and scalar curvature defined by the 4 dimensional metric $g_{(0)}$.

In order for $\langle T_{\mu\nu}\rangle$ to be finite, both divergent terms should vanish. For the background solutions in which the u dependent terms are suppressed, $g_{(2)}$ and $h_{(4)}$ vanish. The logarithmic divergent term vanishes automatically. The vanishing of the $1/\epsilon^2$ term implies the

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4 dimensional Einstein equation

$$R^{(4)}_{\mu\nu} - \frac{1}{2} g_{(0)\mu\nu} R^{(4)} = 0 \tag{4.44}$$

(for $\mu, \nu = +, -, 2, 3$). This is the source-free equation because neither dilaton nor axion was included in (4.41). It is straightforward to repeat the computation of [34] to include contribution of generic matter fields, and, of course, the final result is to give the correct Einstein equation with sources.

We remark that the 4D Einstein equation for the boundary is in general not equivalent to the 5 dimensional Einstein equation with a negative cosmological constant (flux). However, the 4D Einstein equation happens to agree with the 5D Einstein equation for a class of backgrounds including the ones under consideration. In fact, only the (++)-component of the 4D Einstein tensor is nontrivial $G_{++} = -h_{22} - h_{33}$. For SUGRA backgrounds with generic r-dependence in $g(r, x)$, the 5D SUGRA equations correspond to the RG equations in the field theory [35].

This argument is so far incomplete because the energy momentum tensor computed above was based on the validity of the duality. In general the duality may not hold when the Einstein equation is not valid, in which the later is precisely what we want to check. Thus we have to check independently that the VEV of the SYM energy-momentum tensor is indeed given by (4.41). It has been argued in [32] that $\langle T_{\mu\nu} \rangle = 0$ for a free theory. It is of course not the case for our time-dependent SYM. Consider a Yang-Mills theory coupled to fermions living on a base space with a generic metric $\tilde{g}$. The vacuum expectation value of the energy-momentum tensor generically has UV divergences which need to be regularized in a diffeomorphism-invariant way. By a simple power counting, one can see that $\langle T_{\mu\nu} \rangle$ has potentially a quartic, a quadratic and a logarithmic divergence. By dimensional analysis, the quartic divergence is a constant times the cut-off (Planck) scale M_P^4. This is just the cosmological constant contribution from each quantum field. It is well-known that it cancels for a supersymmetric field theory. The quadratic divergence must be of the form

$$(a\tilde{R}_{\mu\nu} + b\tilde{g}_{\mu\nu}\tilde{R})M_P^2 \tag{4.45}$$

with some numerical constant a, b. The precise form may depend on the regularization scheme. However, one should regularize it so that this vacuum energy-momentum tensor is conserved. This implies that it is proportional to the 4D Einstein tensor, so that its effect can be absorbed by renormalizing the 4D Newton constant. The logarithmically divergent term is not of interest for the purpose of this paper. The finite part of $\langle T_{\mu\nu} \rangle$ has been extensively studied in the context of conformal/Weyl anomaly [36]. A more detailed studies of these shall be interesting.

5. Discussion

In this paper we have compared the two-point functions computed at different regimes of the t'Hooft coupling. We find that the two-point function is not protected by nonrenormalization theorem and the SUGRA result is different from the gauge theory result. Since our gauge theory is supersymmetric, it is plausable that there will exist some modified form of nonrenormalization theorem. It is important to establish their existence and to use them to compute quantities that are nonrenormalized. Such quantities will allow one to compare field and SUGRA calculations directly and thus provide a check of the proposed gravity/gauge duality for the time dependent background.

In the usual AdS/CFT correspondence, the scaling symmetry of the AdS background implies that the dual gauge theory is conformal. Given that the conformal symmetry is preserved at the quantum level, this conformal symmetry has been a powerful tool for analysing the field theory and providing valuable understanding of the duality. In our case, let us denote the 4D energy-momentum tensor by $T_{\mu\nu}$. The usual scaling symmetry (3.32) corresponds to the statement that $T^{\mu}_{\mu} = 0$. This is no longer true for us. The Noether current of the new scaling symmetry (3.29) is

$$J^{\mu} \equiv \delta x^{\nu} T^{\mu}_{\nu}, \qquad \delta x^{\underline{\mu}} = a_{\underline{\mu}} x^{\underline{\mu}}, \tag{5.1}$$

where underlined indices are not summed over and $a_{\mu} = (a_{+}, a_{-}, a_{2}, a_{3}) = (0, 2, 1, 1)$. The corresponding conservation law $\nabla_{\mu} J^{\mu} = 0$ then implies that

$$(\nabla_{\mu} \delta x^{\nu}) T^{\mu}_{\nu} = 0. \tag{5.2}$$

To derive it, we have used the energy-momentum conservation law $\nabla_{\mu} T^{\mu}_{\nu} = 0$. It would be interesting to derive the trace anomaly for our SYM theory, which would provide a valuable test to the correspondence we proposed. We expect that some of the techniques for quantum field theory in curved spacetime can be used here and the problem may become tractable at least if one treats h and χ as small perturbations.

Apart from its possible role in holographic duality, by itself the time dependent super Yang-Mills theory introduced in this paper is already a very interesting field theory because of the time-dependent gauge and axion couplings and its untypical scaling symmetry (3.29). The new scaling symmetry suggests us to look for a new renormalization group different from the usual definition associated with a uniform scaling in all dimensions. Comparing it with the scaling symmetry of our supergravity solutions (2.23), we see that this new renormalization group should be one which has a dual interpretation on the supergravity side. Further work is necessary in order to elucidate these aspects in more detail.

The class of solutions which gives a geodesically incomplete spacetime is of interest for the studies of cosmological singularity. From our preliminary analysis above of the two-point functions, it is suggested that the spacetime structure as seen from the gauge theory is different from that seen by the classical gravity solution. In fact from the gauge theory point of view, the spacetime appears to be non-singular. It is important to analyze the quantum gauge theory in greater details (particularly by including interactions) in order to be more confident about this picture and also to learn about how the spacetime singularity is resolved from a spacetime point of view, and to see what kind of interesting structures (e.g. quantum symmetries) may appear on the way.

This paper is only the first step in establishing the connection between time-dependent backgrounds in string theory and gauge theory. There are still many important open questions. For example, as we mentioned at the end of section 3.2, the matching of functional

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parameters for the duality may be corrected by terms of higher order in α'. More importantly, we hope to be able to understand better the quantum properties of the gauge theory and use it to learn about time-dependent processes in string theory.

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